Research Statement: Sam Freedman

1. Introduction

The moduli space of curves $\mathcal{M}_g$ plays a central role in many areas of mathematics from low-dimensional topology to algebraic geometry. For examples, it parametrizes conformal structures, hyperbolic surfaces, and complex curves. The natural setting for studying dynamical systems on $\mathcal{M}_g$ is in vector bundles over it, which encode both initial starting points and flow directions. One key example is the *Hodge bundle* over $\mathcal{M}_g$, which parametrizes all possible genus g *translation surfaces*. Translation surfaces are vital for understanding the dynamical and geometric properties of $\mathcal{M}_g$. My research combines dynamics, algebraic geometry, and other techniques to analyze these flat surfaces and their moduli, deepening our understanding of the topology and geometry of $\mathcal{M}_g$.

Concretely, *translation surfaces* are pairs (X, ω), where X is a Riemann surface and ω is a holomorphic 1-form. The 1-form ω provides a polygonal decomposition of X where parallel opposite edges are identified by translations. Deforming these edges (while preserving parallelism) produces nearby translation surfaces in the *stratum* $\Omega\mathcal{M}_g(\kappa_1, \ldots, \kappa_s)$, the moduli space of genus g translation surfaces with specified cone angle singularities. The union of these strata forms the Hodge bundle over $\mathcal{M}_g$. There is an essential $\mathrm{SL}(2, \mathbb{R})$-action on strata of translation surfaces, where a matrix in $\mathrm{SL}(2, \mathbb{R})$ acts linearly on a polygonal representation of the translation surface. This dynamical action enriches the classical geodesic flow on $\mathcal{M}_g$, corresponding to the diagonal subgroup of $\mathrm{SL}(2, \mathbb{R})$.

My research focuses on deepening our understanding of this $\mathrm{SL}(2, \mathbb{R})$-action, particularly the structure of its orbit closures. In particular, I have studied *periodic points* on closed $\mathrm{SL}(2, \mathbb{R})$-orbits, as well as the monodromy groups of families of translation surfaces over these orbits. This work not only enhances our knowledge of moduli spaces of translation surfaces but also contributes to related fields such as number theory, low-dimensional topology, and computational and algebraic geometry.

In the following sections, I outline future research directions. Section 2 discusses how we will extend our classification of periodic points from Veech surfaces to new affine invariant subvarieties. Section 3 explores computations of monodromy groups of families of translation surfaces and connections to smooth 4-manifolds and the Putman-Wieland conjecture on the homological actions of surface automorphisms. Section 4 presents ongoing work on the isodelaunay decomposition of strata.

2. Periodic Points

Let (X, ω) be a *Veech surface*, that is, a translation surface whose $\mathrm{SL}(2, \mathbb{R})$-orbit is closed in its stratum. A *periodic point* of a Veech surface (X, ω) is a point that has a finite orbit under the affine automorphism group $\mathrm{Aff}^+(X, \omega)$. Examples include the Weierstrass points of a hyperelliptic surface and points of a square torus with rational coordinates. These points have appeared throughout Teichmüller dynamics, from counting sections of Veech fibrations (see Section 2) to solving finite-blocking problems in polygonal billiards (see, e.g., [1, 2, 5]). Exceptional periodic points have even provided evidence for the existence of higher-rank orbit closures before their formal discovery [1].

Question 1. Given a Veech surface (X, ω), what are its periodic points?

2.1. **Algorithm for general Veech surfaces.** Gutkin–Hubert–Schmidt [24] and Eskin–Filip–Wright [16] showed that a non square-tiled Veech surface has *finitely many* periodic points. While other authors had addressed specific Veech surfaces and computed their finite set of periodic points [4, 42], there was no general method for finding the periodic points of an arbitrary Veech surface. Zawad Chowdhury, Samuel Everett, Destine Lee and I then developed a general algorithm in [15] for computing the periodic points on an arbitrary Veech surface; our proof of correctness is a new proof that a non-square-tiled Veech surface has finitely many periodic points:

Theorem A (Chowdhury–Everett–Freedman–Lee [15])**.** *There is an algorithm for computing the periodic points of a non-square-tiled Veech surface.*

Corollary B. *A non-square-tiled Veech surface has finitely many periodic points.*

The algorithm in Theorem A uses techniques from mapping class groups, dynamics, and combinatorics to prune candidate periodic points in stages. First, it finds points periodic under two multi-twists, and uses that (X, ω) is not square-tiled to limit candidates to a set of line segments. Then, it constructs an automorphism $\phi \in \mathrm{Aff}^+(X, \omega)$ that moves the segments transversely, further restricting the candidates to their intersections. Finally, a graph algorithm identifies which intersection points are periodic under all of $\mathrm{Aff}^+(X, \omega)$.

2.2. **Prym eigenforms.** As an application of the algorithm in Theorem A, we analyzed a class of genus 3 Veech surfaces whose periodic points had not yet been classified called *Prym eigenforms* [35]. These surfaces in the stratum $\Omega\mathcal{M}_3(4)$ come with a *Prym involution* that acts as rotation by π on the surface. After testing small complexity examples, we conjectured that the periodic points were exactly the fixed points of the Prym involution. In a second paper [20], I established this conjecture not only for genus 3 Prym eigenforms, but also for the other *minimal* Prym eigenforms in genus 2 and 4; the genus 2 case is a new geometric proof of a theorem of Möller [38].

Theorem C (Freedman [20])**.** *The periodic points of a non square-tiled Prym eigenform in the stratum $\Omega\mathcal{M}_g(2g-2)$ for $2 \leq g \leq 4$ are the fixed points of its Prym involution.*

The proof of Theorem C applies uniformly to all three Prym eigenform genera, following the algorithm in Theorem A. A general lemma uses techniques from mapping class groups to first reduce the search to a finite set of horizontal and vertical saddle connections by classifying points periodic under two multi-twists. To eliminate false candidates, I adapt McMullen's butterfly moves [34], a collection of $\mathrm{GL}(2, \mathbb{R})$-renormalizations that exchange between different periodic directions. These automorphisms displace nonperiodic points in all sufficiently high-complexity eigenform loci, leaving only the fixed points as periodic.

2.3. **Periodic points of other orbit closures.** With both an algorithm (Theorem A) to compute periodic points, and the general theory I developed for analyzing Prym eigenforms in Theorem C, I am in a strong position to classify periodic points on other Veech surfaces and their analogues. For instance, there are Veech surfaces whose periodic points are not yet classified:

Problem 2. Classify the periodic points of the remaining known primitive Veech surfaces.

For example, the Gothic Veech surfaces [17, 36] come with involutions in their defining data, as in the case of Prym eigenforms, so we find it natural to conjecture that their fixed points are again the only periodic points. As for the remaining Bouw–Möller surfaces, I plan to combine our newly established techniques, such as using butterfly moves to find uniform structure in Veech groups, with the previous work of Apisa–Saavedra–Zhang [4] and B. Wright [42] to establish that the periodic points are exactly the fixed points of the hyperelliptic involution.

To tackle these problems, I will also draw from my expertise analyzing non-minimal Prym eigenforms in my work with Boulanger that shows

Theorem D (Boulanger-Freedman [11])**.** *There are no Prym eigenform Veech surfaces in* $\Omega\mathcal{M}_3(2,2)$.

This work required a careful enumeration of prototypical eigenforms in non-minimal Prym strata. Such prototypes might also admit a theory of butterfly moves, leading to admissible cylinder shears essential to ruling out points as being periodic.

Question 3. Are all periodic points of Veech surfaces the fixed points of an automorphism?

In addition to analyzing specific cases, our analysis of the linear-algebraic constraints in [15] has led me to conjecture that well-chosen affine shears are sufficient for classifying periodic points:

Conjecture 1. *If a point on a Veech surface has a finite orbit under three well-chosen independent cylinder shears, then that point should be a periodic point.*

My work in Theorem C verifies this conjecture for Prym eigenforms in genus 2, where three well-chosen butterfly moves are sufficient, and we expect that further arguments could establish it for genus 3 and 4. As cylinder shears are concrete and highly amenable to computation, this would prove a powerful result for classifying periodic points in future cases.

I will also consider the notion of $\mathcal{M}$*-periodic points*, a generalization of periodic points from Veech surfaces to all affine invariant subvarieties $\mathcal{M}$ such as entire strata of translation surfaces.

Problem 4. Classify the $\mathcal{M}$-periodic points of higher-rank affine invariant subvarieties $\mathcal{M}$, such as Prym loci in non-minimal strata.

I propose to classify $\mathcal{M}$-periodic points with an inductive strategy, *degenerating* surfaces in $\mathcal{M}$ to simpler Veech surfaces. This strategy proved effective when Apisa [2] used Möller's [38] determination of periodic points in genus 2 to classify $\mathcal{M}$-periodic points of rank 2 eigenform loci in $\Omega\mathcal{M}_2$. Analogously, I will degenerate Prym eigenforms in non-minimal strata to the minimal Prym eigenforms classified in Theorem C, determining which points of the original eigenforms are $\mathcal{M}$-periodic. I believe that recent advances in compactifications of strata, like the multi-scale differentials compactification [7], will make these degeneration arguments especially effective.

3. Families of Veech surfaces

We will describe a natural construction, originally due to Möller [39], which packages a family of Veech surfaces into a cohesive algebro-geometric structure as in the case of Lefschetz fibrations from 4-manifold theory. They originated as a tool in Teichmüller dynamics for giving a Hodge-theoretic characterization of Teichmüller curves [39], and other others have used them for constructing new Teichmüller curves [12], establishing non-varying properties for sums of Lyapunov exponents [14], and classifying algebraically primitive Teichmüller curves [28, 32, 37]. What is more, periodic points are exactly the holomorphic sections of these families. After describing the construction, we will describe our computations with Lucas [21] of the monodromy of these families, its essential invariant measuring how the fibers change with respect to the topology of the moduli space.

3.1. **Construction of Veech fibrations.** Let (X, ω) be a Veech surface generating a closed $\mathrm{SL}(2,\mathbb{R})$-orbit in its stratum. The image of this closed orbit in moduli space is a geodesically-immersed hyperbolic surface, or a *Teichmüller curve* $C \to \mathcal{M}_g$.

As $\mathcal{M}_g$ and C are in general orbifolds, to build a family of Veech surfaces containing (X, ω) we must first choose a manifold cover $\widetilde{\mathcal{M}}$ of $\mathcal{M}_g$. We can then pull back C to a hyperbolic surface $B \to C$ that maps into the cover $\widetilde{\mathcal{M}}$. Now B has a natural family of Veech surfaces over it, and after passing to a further finite cover, this family extends to a complex surface $\mathbb{X} \to B$.

It is more convenient to work with closed complex surfaces, which we can achieve using the general process of (semi-stable) completion. The resulting family $\widetilde{\mathbb{X}} \to \overline{B}$ is now a *smooth* closed 4-manifold. The space $\widetilde{\mathbb{X}}$ is a fibration where all but finitely many fibers are deformations of the original Veech surface (X, ω), and there finitely many singular fibers over the cusps of B. In analogy with elliptic fibrations, where the generic fiber is an elliptic curve, we call these families *Veech fibrations*.

3.2. **Monodromy for algebraically primitive examples.** Let (X, ω) be a Veech surface with Teichmüller curve C. A natural, concrete class of manifold covers of $\mathcal{M}_g$ are the level m congruence subgroups of the mapping class group $\mathrm{Mod}(X)$, which measure how the mapping class groups acts on homology $H_1(X; \mathbb{Z}/m\mathbb{Z})$. Let $B_m \to C$ be the corresponding cover of hyperbolic surfaces and $\widetilde{\mathbb{X}}_m \to \overline{B_m}$ the *level m congruence Veech fibrations.* Lucas and I used earlier work of Chen-Möller [14] to derive formulas for a variety of topological and complex-geometric invariants of the family, such as its fundamental group and signature, in terms of the cover $B_m \to C$.

Computing the degree of the cover $B_m \to C$ amounts to compute the image of the mod m homology actions $\rho_m : \mathrm{Aff}^+(X, \omega) \to \mathrm{Sp}(H_1(X; \mathbb{Z}/m\mathbb{Z}))$. This task is in general quite difficult, as it relates to computing Kontsevich–Zorich monodromy groups, which are not well-understood (see, e.g., Filip [19, 18] and Matheus–Möller–Yoccoz [31]). But in the case when (X, ω) is algebraically primitive (i.e., has a high-degree trace field) we determined the degree of the congruence covers $B_m \to C$, and we derived topological and complex-geometric information about the fibration $\widetilde{\mathbb{X}}_m$:

Theorem E (Freedman-Lucas [21])**.** *Let (X, ω) be a genus 2 Weierstrass eigenform in $\Omega\mathcal{M}_2(2)$ with nonsquare discriminant [33], an algebraically primitive regular n-gon surface [12], or a sporadic Veech surface E_7 and E_8 [29]. For a prime $p \geq 3$ in an explicit infinite set depending on (X, ω), the image of the monodromy map ρ_m is isomorphic to* $\mathrm{SL}(2, \mathbb{F}_{p^g})$.

The three families in Theorem E comprise all known algebraically primitive Veech surfaces. Arithmetic techniques are key to our arguments, since algebraically primitive Veech surfaces admit real multiplication by their trace field on the entirety of their first homology $H_1(X)$ [39]. This theorem allows us to explicitly compute Chern numbers for many algebraically primitive Veech fibrations, settling them in the geography of all complex surfaces.

3.3. **Families over other affine invariant subvarieties.** Our work with Lucas [21] considered Veech fibrations whose base Teichmüller curves are algebraically primitive. While this family is a rich class of examples, other important Veech surfaces, such as square-tiled surfaces and non-minimal Prym eigenforms in genus 3 and 4, are not algebraically primitive. The primary difficulty is again to determine the image of the monodromy.

Problem 5. Compute the image of the monodromy $\rho_m : \mathrm{Aff}^+(X,\omega) \to \mathrm{Sp}(H_1(X;\mathbb{F}_m))$ for Veech surfaces (X,ω) that are not algebraically primitive.

Analysis of the representation ρ_m in the non algebraically primitive case is more difficult because non-primitive surfaces admit real multiplication on only *part* of $H_1(X)$. But there are techniques from the algebraically primitive case, such as using *Dickson's criterion* [23] to bound the image of ρ_m, that do apply to general Veech fibrations.

3.4. **Putman-Wieland Conjecture.** The Putman–Wieland conjecture [40], equivalent to Ivanov's conjecture on the homology of finite-index subgroups of the mapping class group, predicts that homological actions of surface automorphisms along branched covers are "as complex as possible." While Landesman-Litt [27] proved the conjecture for covers of degree less than g^2, where g is the genus of the base surface, many cases remain open.

Counterexamples from flat geometry arise in genus one, such as the well-known *Eierlegende Wollmilchsau* square-tiled surface, where the homological actions can have large fixed parts. Apisa [3] suggested a rephrasing of the conjecture, showing that any potential counterexample can be expressed as a cover of square-tiled surfaces. The key is then finding new covers $\mathcal{O}' \to \mathcal{O}$ where $\mathrm{Aff}^+(\mathcal{O})$ acts compactly on a large part of the homology, captured by the *Forni subspace* of $H_1(\mathcal{O}';\mathbb{Q})$.

Problem 6. Find new examples of covers of square-tiled surfaces whose Forni subspaces are nontrivial, establishing new counterexamples or cases of the Putman–Wieland conjecture.

A quantitative version of this problem appears in Landesman–Litt [26], where they ask for the distribution of dimensions of Forni subspaces across families of square-tiled surfaces. These statistics could help make more precise predictions about the nature of Forni subspaces. We will leverage our past experience [21] computing monodromy groups and computational methods like the Flatsurf software package to address the case of Forni subspaces of square-tiled surfaces. These computations could shed new light on the remaining cases of the Putman–Wieland conjecture.

4. Isodelaunay Decomposition of Strata of Translation Surfaces

The topology of strata of translation surfaces is still largely mysterious. While Kontsevich [25] conjectured that strata are $K(\pi,1)$-spaces, this has been verified only for strata of low complexity [30]. One concrete approach for studying the homology of strata would be to decompose them into a cell complex. Similar to how $\mathcal{M}_g$ has a cell complex based on the combinatorics of ribbon graphs, we can partition translation surfaces by the combinatorial type of their *Delaunay decomposition.* This Delaunay decomposition, generically a triangulation, is canonically defined and features polygons that maximize the minimal angle across the decomposition. This makes Delaunay triangulations crucial tools in mathematical modeling, numerical analysis, and other applied disciplines.

Little is known about these regions in the strata, including whether they are connected or are even cells (i.e., contractible), partly due to the complexity of the quartic equations defining them. In work-in-progress with Zykoski, computations and heuristics from low-complexity cases suggest the following conjecture:

Conjecture 2. *The isodelaunay cells in strata of translation surfaces are contractible.*

Our approach, inspired by the work of Rivin [41] and Bobenko-Springborn [9], is to establish convexity for natural functions defined on isodelaunay cells, such as *harmonic index* $H(\tau) = \sum_{T\in\tau}\sum_{e\in T}\ell(e)^2/\mathrm{Area}(T)$ which measures how far from equilateral the polygons are. As these gradient flows tend to keep polygons from being too thin, they stay away from the boundaries of their regions and tend to our conjectural critical points. In fact, computer simulation of these gradient flow suggests that the extract structure of the critical point in many cases is of a sheared square-tiled surface.

With these Delaunay cells in hand, we will then analyze the geometric and homological properties of this decomposition:

Problem 7. Enumerate all the possible isodelaunay cells in each strata. Show that the intersections of multiple cells are themselves contractible. Compute the cellular homology groups of strata using the isodelaunay cell complex

Our work [15] on periodic points used Delaunay triangulations in an essential way, so we expect to leverage this experience. After solving these problems, we can use Delaunay decompositions to code $\mathrm{SL}(2,\mathbb{R})$-orbits according to the regions they visit, in comparison with earlier coding schemes like Rauzy-Veech diagrams (see, e.g., [6, 8, 10]). This coding would gives a new combinatorial approach to studying the monodromy representation for the fundamental group of a stratum, an area of considerable interest [13]. It could also shed light too on the monodromy kernel [22], which has not yet been geometrically understood in the strata. Moreover, precise understanding of the Delaunay cells, their number, and their relationships could lead to verification of the Kontsevich conjecture for new components of strata, bounds on the rank of cohomology groups, and more.

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