RESEARCH STATEMENT – SAM FREEDMAN

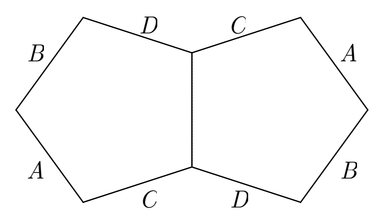
1. Introduction

I study *Veech surfaces*: Riemann surfaces with exceptionally symmetric flat structures that generalize flat tori. They play a crucial role in understanding moduli spaces of flat surfaces, and their study has led to developments in number theory, the dynamics of rational billiards, and algebro-geometric properties of the moduli space $\mathcal{M}_g$ of genus g Riemann surfaces. My research has furthered our understanding of Veech surfaces from a range of perspectives, including dynamical systems (*periodic points* of Veech surfaces) and complex algebraic geometry (*families* of Veech surfaces).

After giving some background, I will describe how I will extend my previous work on classifying periodic points to a variety of new Veech surfaces. Then I will discuss my progress on investigating the geometry and topology of Veech fibrations, and how I will situate these families in the geography of all complex surfaces.

2. Background

A *translation surface* (X, ω) is Riemann surface X with a holomorphic 1-form ω; the surface can be presented as a union of Euclidean polygons with parallel opposite edges identified by translations. Deforming the edges (in a way preserving parallel sides) produces neighboring translation surfaces in the same *stratum* $\Omega\mathcal{M}_g(\kappa_1, \dots, \kappa_s)$ of genus g translation surfaces with cone angles $(\kappa_i + 1)2\pi$ at the vertices. The strata assemble together into a bundle $\Omega\mathcal{M}_g$ over the moduli space $\mathcal{M}_g$ of genus g Riemann surfaces.



Translation surfaces can admit *affine automorphisms* that preserve the flat structure of ω. These automorphisms have a well-defined linear part, giving a map $D : \mathrm{Aff}^+(X, \omega) \to \mathrm{SL}(2, \mathbb{R})$. The collection of all linear parts $D(\mathrm{Aff}^+(X, \omega))$ is the *Veech group* $\mathrm{SL}(X, \omega)$.

While most translation surfaces have a trivial Veech group [33], there are rare examples with an exceptional number of symmetries in the sense that $\mathrm{SL}(X, \omega)$ is a *lattice* in $\mathrm{SL}(2, \mathbb{R})$. These highly symmetric translation surfaces are *Veech surfaces*. The prototypical example of a Veech surface is a flat torus; we so far know of five infinite families and three sporadic examples [29].

Translation surfaces have an $\mathrm{SL}(2, \mathbb{R})$-action: a matrix A acts on (X, ω) by applying A linearly to each polygon. Through deep theorems of Eskin–Mirzakhani [13], Eskin–Mirzakhani–Mohammadi [14], and Filip [18], all $\mathrm{SL}(2, \mathbb{R})$ orbit closures are *affine invariant subvarieties (AISs)*, well-behaved geometric objects that are locally linear. When (X, ω) is Veech, the orbit $\mathrm{SL}(2, \mathbb{R}) \cdot (X, \omega)$ is a closed subvariety [37]; the projection of this closed orbit from $\Omega\mathcal{M}_g$ to $\mathcal{M}_g$ is a *Teichmüller curve* C in $\mathcal{M}_g$.

3. Periodic Points of Veech surfaces

A *periodic point* of a Veech surface (X, ω) is a point that has a finite orbit under the affine automorphism group $\mathrm{Aff}^+(X, \omega)$. Examples include the zeros of ω, the 6 Weierstrass points of a genus 2 translation surface, and points of a square torus with rational coordinates. These points have appeared throughout Teichmüller dynamics, from counting sections of Veech fibrations to solving finite-blocking problems in polygonal billiards (see, e.g., [2, 4, 1]). Exceptional periodic points have even predicted the existence of higher-rank orbit closures before they were formally discovered [1].

In light of their prevalence and usefulness, we ask:

Question 1. Given a Veech surface (X, ω), what are its periodic points?

3.1. **Algorithm for general Veech surfaces.** Gutkin–Hubert–Schmidt [23] and Eskin–Filip–Wright [12] showed that a non square-tiled Veech surface has *finitely many* periodic points. While other authors had addressed specific Veech surfaces and computed their finite set of periodic points [3, 38], there was no general method for finding the periodic points of an arbitrary Veech surface. Zawad Chowdhury, Samuel Everett, Destine Lee and I then developed a general algorithm in [11] for computing the periodic points on an arbitrary Veech surface; our proof of correctness is a new proof that a non-square-tiled Veech surface has finitely many periodic points:

Theorem 1 (Chowdhury–Everett–Freedman–Lee [11])**.** *There exists an algorithm that, given a non-square-tiled Veech surface as input, outputs the periodic points on that translation surface.*

Corollary 1. *A non-square-tiled Veech surface has finitely many periodic points.*

The algorithm in Theorem 1 prunes the set of candidate periodic points in three stages:

(1) It finds the points periodic under two *multi-twists*, constraining candidates to a finite set of line segments. (This step uses that (X, ω) is not square-tiled.) These line segments arise from the fact that a periodic point must have a rational height in each cylinder containing it; when a point is contained in three generic cylinders, the three rational conditions combine to determine a linear constraint.
(2) It constructs an automorphism $\phi \in \mathrm{Aff}^+(X, \omega)$ that moves the set of line segments transverse to itself; this restricts the candidates to the finite set of intersection points.
(3) It uses a well-known graph algorithm to determine which intersection points are periodic under all of $\mathrm{Aff}^+(X, \omega)$.

3.2. **Prym eigenforms.** As an application of the algorithm in Theorem 1, we analyzed genus 3 *Prym eigenform* Veech surfaces [31] with a single cone point whose periodic points had not yet been classified. After testing small complexity examples, we conjectured that the periodic points were the fixed points of a certain *Prym involution*. In a second paper [20], I established this conjecture not only for genus 3 Prym eigenforms, but also for the other *minimal* Prym eigenforms in genus 2 and 4; the genus 2 case is a new geometric proof of a theorem of Möller [35].

Theorem 2 (Freedman [20])**.** *The periodic points of a non square-tiled Prym eigenform in the stratum $\Omega\mathcal{M}_g(2g-2)$ for $2 \leq g \leq 4$ are the fixed points of its Prym involution.*

The proof of Theorem 2 treats all three genera of Prym eigenforms in essentially the same manner, shadowing the algorithm of Theorem 1. I start by proving a general lemma that classifies the points interior to a horizontal and vertical cylinder that have a finite orbit under the two multi-twists. This reduces the search space to a finite set of horizontal and vertical saddle connections at the boundary of the cylinders. In the spirit of step (2) above, it remains to construct affine automorphisms that move the false candidates to nonperiodic points. For this I repurpose McMullen's *butterfly moves*: a collection of $\mathrm{GL}(2, \mathbb{R})$-renormalizations of the eigenform that exchange between different periodic directions. For eigenform loci of sufficiently large complexity, I find surfaces that admit enough butterfly moves to displace the candidate points, proving that only the fixed points are periodic. I then rule out the nonfixed candidate points of eigenforms in the finitely many remaining loci using our algorithm in Theorem 1.

3.3. **Periodic points of other affine invariant subvarieties.** With both an algorithm (Theorem 1) to compute periodic points, and the general theory I developed for Theorem 2 on Prym eigenforms, I am in a strong position to classify periodic points on other affine invariant subvarieties. For instance, there are Veech surfaces whose periodic points are not yet classified:

Problem 2. Determine the periodic points of the remaining unclassified Veech surfaces: the Gothic Veech surfaces, the remaining Bouw–Möller surfaces, and the three sporadic E_6, E_7 and E_8 examples.

The Gothic Veech surfaces [15, 32] also come with involutions, mirroring Prym eigenforms, so there are natural candidate periodic points to test with our algorithm. As for the remaining Bouw–Möller surfaces, we plan to combine our newly established techniques with the previous work of Apisa–Saavedra–Zhang [3] and B. Wright [38] on the $(2, n)$ and $(3, n)$ cases respectively.

We will also consider the notion of $\mathcal{M}$*-periodic points* that generalize periodic points from Veech surfaces to other affine invariant subvarieties $\mathcal{M}$:

Problem 3. Classify the $\mathcal{M}$-periodic points of higher-rank affine invariant subvarieties $\mathcal{M}$, such as Prym loci in nonminimal strata.

We propose to classify $\mathcal{M}$-periodic points by *degenerating* surfaces in $\mathcal{M}$ to simpler Veech surfaces. This strategy proved effective when Apisa [2] used Möller's [35] determination of periodic points in genus 2 to classify $\mathcal{M}$-periodic points of rank 2 eigenform loci in $\Omega\mathcal{M}_2$. Analogously, we will degenerate Prym eigenforms in nonminimal strata to the minimal Prym eigenforms classified in Theorem 2, determining which points of the original eigenforms are $\mathcal{M}$-periodic. I will draw from my expertise analyzing nonminimal Prym eigenforms in my work with Julian Boulanger [6] that shows there are no Prym eigenform Veech surfaces in $\Omega\mathcal{M}_3(2, 2)$.

4. Families of Veech surfaces

Let (X, ω) be a Veech surface with Teichmüller curve $C \to \mathcal{M}_g$. Every torsion free, finite index subgroup of the mapping class group $\mathrm{Mod}(X)$ determines a manifold cover $\widetilde{\mathcal{M}}$ of $\mathcal{M}_g$ and a finite covering $B \to C$ where B is a hyperbolic surface. Pulling back the universal family over $\widetilde{\mathcal{M}}$ to B, and possibly passing to a finite cover, we produce a complex surface $\mathbb{X} \to B$ whose (semi-stable) completion $\widetilde{\mathbb{X}} \to \overline{B}$ is a smooth 4-manifold. The manifold $\widetilde{\mathbb{X}}$ is a four real-dimensional fibration with generic fiber a Veech surface; the exceptional fibers corresponding to the cusps of B are singular with nodal singularities. In analogy with elliptic fibrations, where the generic fiber is an elliptic curve, we call these families *Veech fibrations*.

Veech fibrations originated as a tool in Teichmüller dynamics when Möller [36] used them to give a Hodge-theoretic characterization of Teichmüller curves. Other authors have used them for constructing new Teichmüller curves [7], establishing non-varying properties for sums of Lyapunov exponents [10], and classifying algebraically primitive Teichmüller curves [34, 28, 24].

Because of their widespread usefulness and concrete presentation in terms of Veech surfaces, Veech fibrations are a natural class of smooth 4-manifolds for study further. To understand where Veech fibrations sit in the geography of all complex surfaces, we must first address:

Question 4. Given a Veech fibration $\widetilde{\mathbb{X}} \to \overline{B}$, how can we compute its topological invariants, such as its fundamental group $\pi_1(X)$, and its complex-geometric invariants, such as its Chern numbers $c_1^2(\widetilde{\mathbb{X}})$ and $c_2(\widetilde{\mathbb{X}})$?

In the next subsection I will answer this question for the important class of *algebraically primitive* Veech fibrations.

4.1. Algebraically primitive examples.

Let $\widetilde{\mathbb{X}} \to \overline{B}$ be a Veech fibration with fiber (X, ω) and Teichmüller curve C. Towards Question 4, Trent Lucas and I derived formulas for a variety of invariants of $\widetilde{\mathbb{X}} \to \overline{B}$, including the fundamental group, Betti numbers, Euler characteristic, and signature, using earlier work of Chen–Möller [10]. Our explicit formulas depend on explicitly computing the degree of the cover $B \to C$, a task that is difficult for an arbitrary Veech fibration.

We focused on the *level m congruence Veech fibrations* $\widetilde{\mathbb{X}}_m \to \overline{B_m}$, those corresponding to the level m congruence subgroups of the mapping class group $\mathrm{Mod}(X)$. Computing the degree of the cover $B_m \to C$ in this situation amounts to compute the image of the mod m homology actions

$$\rho_m : \mathrm{Aff}^+(X,\omega) \to \mathrm{Sp}(H_1(X;\mathbb{Z}/m\mathbb{Z})).$$

This task is also difficult, as it relates to computing Kontsevich–Zorich monodromy groups, and these are not well-understood (see, e.g., Filip [19, 17] and Matheus–Möller–Yoccoz [27]). But in the case when (X,ω) is *algebraically primitive*, we determined the degree of the congruence covers $B_m \to C$, and we derived topological and complex-geometric information about the Veech fibration $\widetilde{\mathbb{X}}_m$:

Theorem 3 (Freedman–Lucas [21]). *Let (X,ω) be a genus g algebraically primitive Veech surface in one of the following families:*

1. *the genus 2 Weierstrass eigenforms in $\Omega\mathcal{M}_2(2)$ with nonsquare discriminant [30],*
2. *the regular n-gon surfaces with n a prime, twice a prime or a power of 2 [7], or*
3. *the sporadic Veech surfaces E_7 and E_8 [26].*

For a prime $p \geq 3$ in an explicit infinite set depending on (X,ω), the degree of the congruence cover $B_p \to C$ is $|\,\mathrm{SL}(2,\mathbb{F}_{p^g})|$. Moreover, the natural map $\pi_1(\widetilde{\mathbb{X}}_p) \to \pi_1(\overline{B_p})$ is an isomorphism and $\widetilde{\mathbb{X}}_p$ is a minimal complex surface of general type.

The three families in Theorem 3 comprise all known algebraically primitive Veech surfaces. Key to our arguments is that algebraically primitive surfaces admit *real multiplication* by their *trace field* on the entirety of their first homology $H_1(X)$ [36].

We can use Theorem 3 to compute the Chern numbers for any explicit example of an algebraically primitive Veech fibration. For example, we showed:

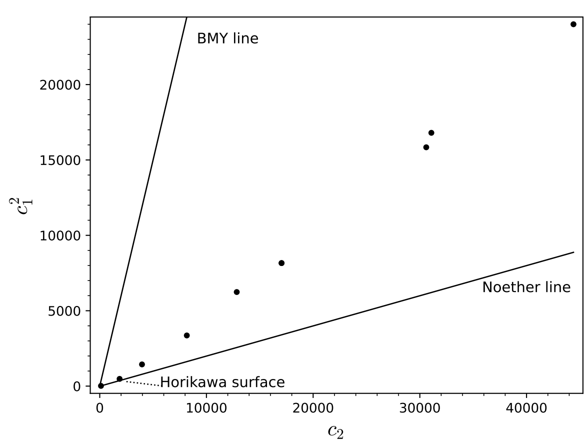
Theorem 4 (Freedman–Lucas [21]). *Let (Y_q,ω_q) be the double regular q-gon surface of genus $g = (q-1)/2$ where $q \geq 5$ is prime. Fix $p \geq 3$ a prime such that the minimal polynomial of $4\cos(\pi/q)^2$ is irreducible over $\mathbb{F}_p$, and let $\widetilde{\mathbb{X}}_{q,p} \to \overline{B}_{q,p}$ be the level p congruence Veech fibration. Then*

$$c_2(\widetilde{\mathbb{X}}_{q,p}) = dg + d(q-3)\left(\frac{1}{2} - \frac{1}{q} - \frac{1}{p}\right), \quad \textit{and} \quad c_1^2(\widetilde{\mathbb{X}}_{q,p}) = 2c_2(\widetilde{\mathbb{X}}_{q,p}) - \frac{3d(q^2-1)}{4q}.$$

The table below shows Chern numbers of level 3 Veech fibrations built from Weierstrass eigenforms in family (1). Upon examining this data, Lucas and I discovered that *Horikawa surfaces*, a classified family of general type complex surfaces, can arise as Veech fibrations:

Corollary 2 (Freedman–Lucas [21]). *The level 3 congruence Veech fibration $\widetilde{\mathbb{X}}_3 \to B_3$ of the double regular pentagon surface is isomorphic to a Horikawa surface.*

The double pentagon Veech fibration is among the simplest Horikawa surfaces and has been well-studied [5].



4.2. **Families over other affine invariant subvarieties.** Our work with Lucas [21] on Question 4 considered Veech fibrations whose base Teichmüller curves are algebraically primitive. While this family is a rich class of examples, other important Veech surfaces, such as *square-tiled surfaces* and genus 3 and 4 *Prym eigenforms*, are not algebraically primitive. To establish which complex surfaces can arise in full generality, we must compute these other fundamental groups, Chern numbers, and Kodaira dimensions. The primary difficulty is to:

Problem 5. Compute the image of the representation $\rho_m : \mathrm{Aff}^+(X, \omega) \to \mathrm{Sp}(H_1(X; \mathbb{F}_m))$ for Veech surfaces (X, ω) that are not algebraically primitive.

Analysis of the representation ρ_m in the non algebraically primitive case is more difficult because non-primitive surfaces admit real multiplication on only *part* of $H_1(X)$. But there are techniques from the algebraically primitive case, such as using *Dickson's criterion* [22] to bound the image of ρ_m, that do apply to general Veech fibrations.

Apart from Teichmüller curves, there are *higher-rank* affine invariant subvarieties in $\mathcal{M}_g$, such as loci of genus 2 Weierstrass eigenforms with a fixed discriminant, that have not yet been studied from the modular point of view. We propose to apply our methods for studying Veech fibrations to this much broader class of complex-geometric spaces:

Problem 6. Compute the Betti numbers, Chern numbers and Kodaira dimension of Veech fibrations constructed from higher-rank affine invariant subvarieties.

The analogous representations ρ_m for higher-rank affine invariant subvarieties connect to recent work describing the topology of strata of translation surfaces via their monodromy groups [8, 9]. Just as for Teichmüller curves, I expect that we can compute the higher Chern numbers of these families in terms of the monodromy groups.

4.3. **Mapping class groups.** *Mapping class groups* are algebraic invariants that encode homeomorphisms of a manifold up to deformation; they are central to the study of real surfaces and the moduli space $\mathcal{M}_g$ of genus g Riemann surfaces. In parallel to these developments, authors have considered mapping class groups of complex surfaces, seeking to gain similar insights for 4-dimension manifolds [16, 25]. Because we can understand Veech fibrations hands-on through the flat geometry of Veech surfaces, we can "see" their symmetries and look for structure in their mapping class groups. As a first step, I propose to:

Problem 7. Find explicit representatives of mapping classes of Veech fibrations.

For instance, elements of the translation group $\mathrm{Aut}(X, \omega)$ act as translations on the fibers and could give fiber-wise automorphisms. In another direction, when there is an isomorphism $\pi_1(\widetilde{\mathbb{X}}) \cong \pi_1(\overline{B})$, as for the algebraically examples in Theorem 3, the Dehn–Nielsen–Behr theorem yields a well-defined homomorphism

$$\phi : \mathrm{Mod}(\widetilde{\mathbb{X}}) \to \mathrm{Out}(\pi_1(\widetilde{\mathbb{X}})) \xrightarrow{\cong} \mathrm{Out}(\pi_1(\overline{B})) \xrightarrow{\cong} \mathrm{Mod}(\overline{B}).$$

To use the well-known structure of mapping class group of the real surface $\overline{B}$, we must establish:

Problem 8. For which Veech fibrations $\widetilde{\mathbb{X}} \to \overline{B}$ is the map $\phi : \mathrm{Mod}(\widetilde{\mathbb{X}}) \to \mathrm{Mod}(\overline{B})$ surjective or an isomorphism?

I expect that main challenge will be to determine which mapping class of $\widetilde{\mathbb{X}}$ can be deformed to preserve each of the Veech surface fibers set-wise.

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